SEMINAR NR. 14, REZOLVĂRI
Algebră liniară şi Geometrie analitică

13. **CUADRICE ÎN SPAŢIU** (în $\mathbb{R}^3/\mathbb{V}_3$)

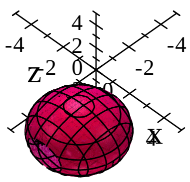
Denumirea	Exemple
elipsoid, sferă	$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$ $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$
cuadrica vidă (elipsoid imaginar)	$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=-1$
punct dublu (x_0,y_0,z_0)	$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=0$
cilindru eliptic	$\begin{cases}\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1\\ z\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1\\ x\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(x-x_0)^2}{a^2}+\frac{(z-z_0)^2}{c^2}=1\\ y\in\mathbb{R}\end{cases}$
dreapta dublă paralelă cu $Oz/Ox/Oy$	$\begin{cases}\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=0\\ z\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=0\\ x\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(x-x_0)^2}{a^2}+\frac{(z-z_0)^2}{c^2}=0\\ y\in\mathbb{R}\end{cases}$
planul dublu paralel cu planul $yOz/zOx/xOy$	$\begin{cases}\frac{(x-x_0)^2}{a^2}=0\\ y\in\mathbb{R}\\ z\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(z-z_0)^2}{c^2}=0\\ z\in\mathbb{R}\\ x\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(z-z_0)^2}{c^2}=0\\ x\in\mathbb{R}\\ y\in\mathbb{R}\end{cases}$

hiperboloid cu o pânză	$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1,$ $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1$ $-\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$
hiperboloid cu două pânze	$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1,$ $-\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$ $-\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1$
con pătratic	$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=0,$ $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=0$ $-\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=0$
cilindru hiperbolic	$\begin{cases}\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1\\ z\in\mathbb{R}\end{cases}$, $\begin{cases}-\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1\\ z\in\mathbb{R}\end{cases}$ $\begin{cases}\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1\\ x\in\mathbb{R}\end{cases}$, ş.a $\begin{cases}xy=\pm a^2\\ z\in\mathbb{R}\end{cases}$ ş.a.
reuniunea a două plane secante	$\begin{cases}\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=0\\ z\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=0\\ x\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(x-x_0)^2}{a^2}-\frac{(z-z_0)^2}{c^2}=0\\ y\in\mathbb{R}\end{cases}$

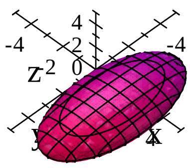
paraboloid eliptic	$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=2(z-z_0)$, $\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=2(x-x_0)$ $\frac{(x-x_0)^2}{a^2}+\frac{(z-z_0)^2}{c^2}=2(y-y_0)$
paraboloid hiperbolic	$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=2(z-z_0)$, $\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=2(x-x_0)$ $\frac{(x-x_0)^2}{a^2}-\frac{(z-z_0)^2}{c^2}=2(y-y_0)$
cilindru parabolic	$\begin{cases}\frac{(x-x_0)^2}{a^2}=2(z-z_0)\\ y\in\mathbb{R}\end{cases}$,ş.a.m.d
reuniune de plane paralele	$\begin{cases}\frac{(x-x_0)^2}{a^2}=1\\ y\in\mathbb{R},\ z\in\mathbb{R}\end{cases}$,ş.a.m.d
reuniune de plane confundate	$\begin{cases}\frac{(x-x_0)^2}{a^2}=0\\ y\in\mathbb{R},\ z\in\mathbb{R}\end{cases}$,
cuadrica vidă	$\begin{cases}x^2=-1\\ y\in\mathbb{R},\ z\in\mathbb{R}\end{cases}$, $\begin{cases}\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=-1\\ z\in\mathbb{R}\end{cases}$,ş.a.m.d

Exemple de cuadrice în spaţiu

a) $(x-3)^2+(y-2)^2+z^2=5$ este ecuaţia sferei cu centrul $(3,2,0)$ şi cu raza $r=\sqrt{5}$.



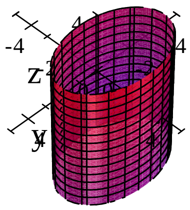
b) $\frac{(x-1)^2}{4^2}+\frac{(y-2)^2}{2^2}+\frac{z^2}{1^2}=1$ este ecuaţia elipsoidului cu centrul $(1,2,1)$ şi cu semiaxele $a=4$, $b=2$, $c=1$.



c) $\frac{(x-1)^2}{3^2}+\frac{(y-2)^2}{2^2}+\frac{(z-2)^2}{2^2}=-1$ este ecuaţia unui elipsoid imaginar (cuadrică vidă).

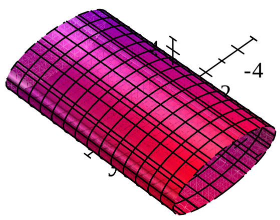
d) $\frac{(x-3)^2}{10}+\frac{(y-1)^2}{5}+\frac{(z-2)^2}{7}=0$ este ecuaţia punctului dublu $(x,y,z)=(3,1,2)$ (este cuadrică degenerată).

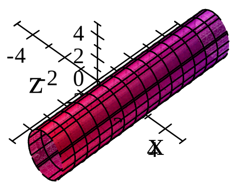
e) $\begin{cases} \dfrac{(x-1)^2}{3^2} + \dfrac{(y-2)^2}{2^2} = 1 \\ z \in \mathbb{R} \end{cases}$ este ecuaţia unui cilindru eliptic.



Cilindri eliptici sunt şi

$\begin{cases} \dfrac{(x-2)^2}{3^2} + \dfrac{(z-1)^2}{2^2} = 1 \\ y \in \mathbb{R} \end{cases}$; $\begin{cases} \dfrac{(y-2)^2}{1^2} + \dfrac{(z-1)^2}{2^2} = 1 \\ x \in \mathbb{R} \end{cases}$.

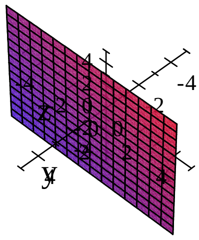




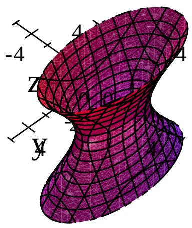
f) $\begin{cases} \dfrac{(x-1)^2}{3^2} + \dfrac{(y-2)^2}{2^2} = 0 \\ z \in \mathbb{R} \end{cases}$ este ecuaţia unei drepte duble paralele cu Oz, anume

$\begin{cases} x = 1 \\ y = 2 \quad , \alpha \in \mathbb{R} \\ z = \alpha \end{cases}$

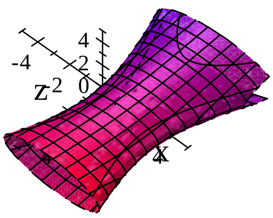
g) $\begin{cases} \dfrac{(x-1)^2}{2} = 0 \\ y \in \mathbb{R} \\ z \in \mathbb{R} \end{cases}$ este ecuaţia unui plan dublu paralel cu planul yOz, anume $x = 1$.

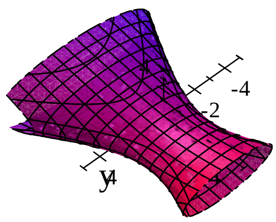


h) $\dfrac{(x-1)^2}{2^2}+\dfrac{(y-2)^2}{1^2}-\dfrac{z^2}{3^2}=1$ este ecuaţia hiperboloidului cu o pânză cu centrul $(1,2,0)$, cu $a=2$, $b=1$, $c=3$.

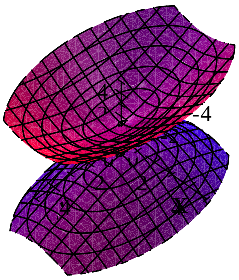


Hiperboloizi cu o pânza sunt şi
$-\dfrac{(x-1)^2}{4^2}+\dfrac{(y-2)^2}{2^2}+\dfrac{z^2}{1^2}=1$, $\dfrac{(x-1)^2}{2^2}-\dfrac{(y-2)^2}{3^2}+\dfrac{z^2}{1^2}=1$.

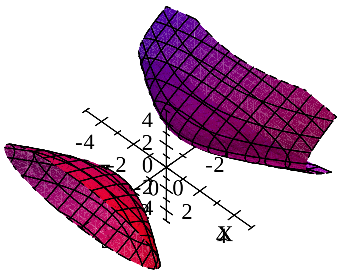


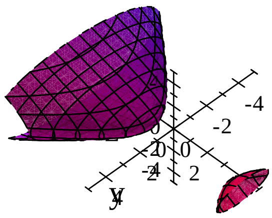


i) $-\dfrac{x^2}{3}-\dfrac{y^2}{1}+\dfrac{z^2}{2}=1$ este ecuaţia hiperboloidului cu două pânze cu centrul $(0,0,0)$, cu $a=\sqrt{3}$, $b=1$ $c=\sqrt{2}$.

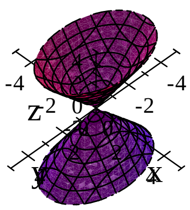


Hiperboloizi cu două pânze sunt şi
$\frac{(x-1)^2}{3} - \frac{y^2}{5} - \frac{(z-2)^2}{1} = 1, -\frac{(x-1)^2}{3} + \frac{(y-2)^2}{5} - \frac{(z-1)^2}{1} = 1.$

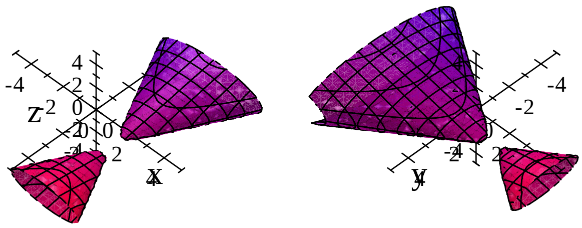




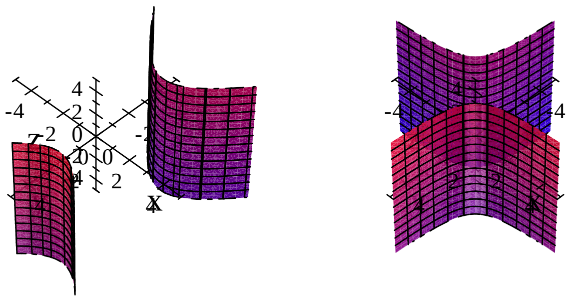
j) $\frac{x^2}{3^2} + \frac{y^2}{2^2} - \frac{z^2}{5^2} = 0$ este con pătratic.



Conuri pătratice sunt şi
$-\frac{(x-1)^2}{4^2} + \frac{(y-2)^2}{2^2} + \frac{z^2}{1^2} = 0, \frac{(x-1)^2}{2^2} - \frac{(y-2)^2}{3^2} + \frac{z^2}{1^2} = 0.$

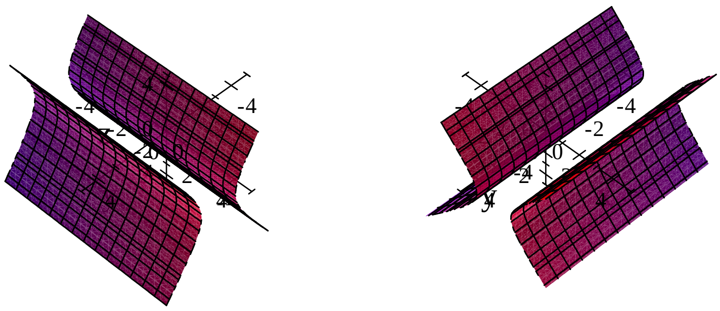


k) $\begin{cases} \dfrac{(x-1)^2}{3^2} - \dfrac{(y-2)^2}{2^2} = 1 \\ z \in \mathbb{R} \end{cases}$; $\begin{cases} xy = 1 \\ z \in \mathbb{R} \end{cases}$ sunt ecuaţii ale unor cilindri hiperbolici.

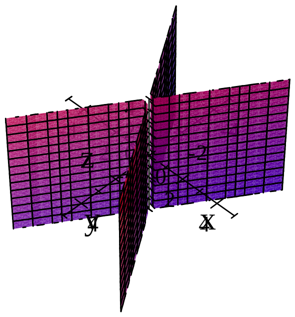


Cilindri hiperbolici sunt şi

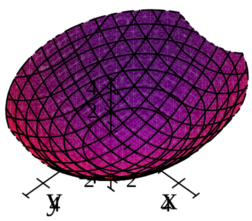
$\begin{cases} \dfrac{(x-2)^2}{1^2} - \dfrac{(z-1)^2}{2^2} = 1 \\ y \in \mathbb{R} \end{cases}$; $\begin{cases} \dfrac{(y-2)^2}{1} - \dfrac{(z-1)^2}{2} = 1 \\ x \in \mathbb{R} \end{cases}$; .



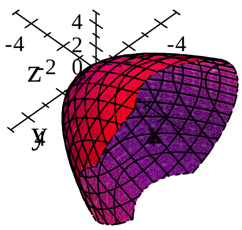
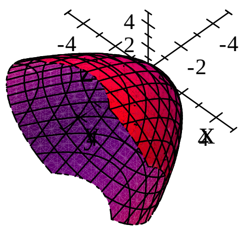
l) $\begin{cases} \dfrac{x^2}{3^2} - \dfrac{y^2}{2^2} = 0 \\ z \in \mathbb{R} \end{cases}$ este ecuaţia perechii de plane $\dfrac{x}{3} + \dfrac{y}{2} = 0$, $\dfrac{x}{3} - \dfrac{y}{2} = 0$, care se intersectează (secante) în dreapta $\begin{cases} x = 0 \\ y = 0 \\ z = \alpha \end{cases}$, $\alpha \in \mathbb{R}$ (e cuadrică degenerată).



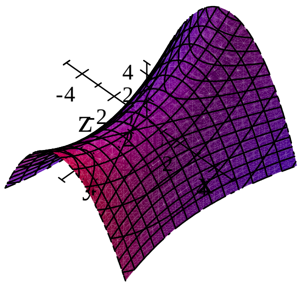
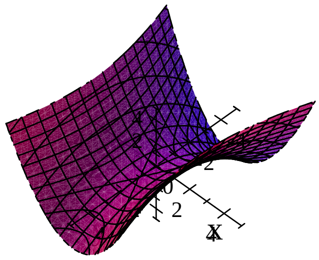
m) $\dfrac{x^2}{3} + \dfrac{(y-1)^2}{2} = 2z$ este ecuaţia unui paraboloid eliptic



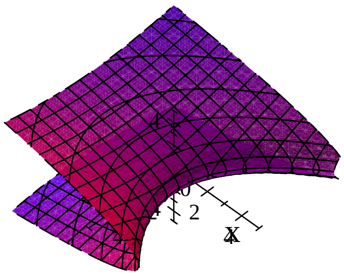
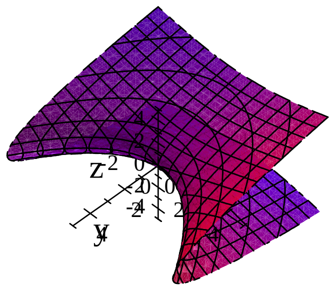
Paraboloizi eliptici sunt şi
$\dfrac{(x-1)^2}{2} + \dfrac{z^2}{3} = 2y$, $\dfrac{(y-1)^2}{2} + \dfrac{z^2}{3} = 2x$.

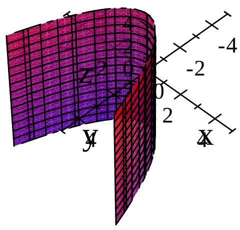
n) $\dfrac{(x-1)^2}{4} - \dfrac{y^2}{1} = 2z, -\dfrac{(x-1)^2}{4} + \dfrac{y^2}{1} = 2z$ sunt ecuaţii ale unor paraboloizi hiperbolici (şa).

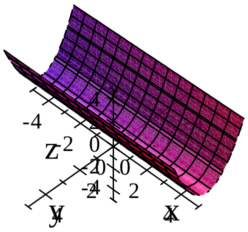
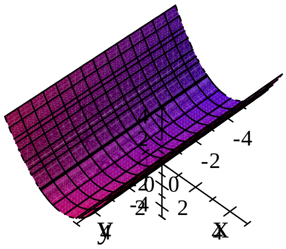
Paraboloizi hiperbolici sunt şi
$\frac{(x-1)^2}{4} - \frac{z^2}{1} = 2\,(y-1)\,, \frac{y^2}{3} - \frac{z^2}{1} = 2x.$

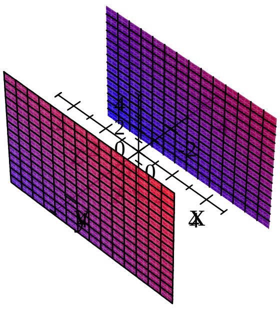
o) $\begin{cases} y^2 = 2x \\ z \in \mathbb{R} \end{cases}$ este ecuaţia unui cilindru parabolic.



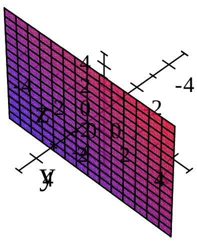
Cilindri parabolici sunt şi
$\begin{cases} z = x^2 + x + 1 \\ y \in \mathbb{R} \end{cases} ; \begin{cases} (y+1)^2 = 2z \\ x \in \mathbb{R} \end{cases} .$

p) $\begin{cases} \dfrac{x^2}{3^2} = 1 \\ y \in \mathbb{R} \\ z \in \mathbb{R} \end{cases}$ este ecuaţia reuniunii de plane $\dfrac{x}{3} = 1$, $\dfrac{x}{3} = -1$, care sunt paralele.



q) $\begin{cases} (x-1)^2 = 0 \\ y \in \mathbb{R} \\ z \in \mathbb{R} \end{cases}$ ecuaţia reuniunii de plane $x = 1$ confundate.



l) $\begin{cases} x^2 = -1 \\ y \in \mathbb{R} \\ z \in \mathbb{R} \end{cases}$, $\begin{cases} x^2 + \dfrac{y^2}{3^2} = -1 \\ z \in \mathbb{R} \end{cases}$ sunt cuadrice vide.