Department of Computer Science, University of Verona
Degree in Applied Mathematics
Probability, Tuesday, June 28^{th}, 2016

Exercises

1. (a) If n students are in a classroom, what is the probability that at least two of them celebrate their birthday on the same day of the year? (we suppose that the year has 365 days)

   Write the associated probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

   (b) Suppose that we have an unbalanced coin which comes up heads with probability $p \in (0,1)$. Find the distribution of the random variable X whose values are the total number of the failures needed to obtain the first head. Compute the mean and the variance of X.

2. Let $X \sim \mathcal{P}(\lambda)$, where $\lambda > 0$, and a sequence of independent random variables $(X_k)_{k\in\mathbb{N}^*}$ such that $X_k \sim \mathcal{P}(1)$.

   (a) Find the mean and the variance of X.

   (b) Find the type of the distribution of the random variable $\bar{X}_n := \dfrac{1}{n}\sum_{k=1}^{n} X_k$.

   Hint: the notation $X \sim \mathcal{P}(\lambda)$ means that X is Poisson distributed with parameter λ, i.e. $\mathbb{P}(X = k) = \frac{\lambda^k}{k!}e^{-\lambda}$.

3. Let us consider a random vector (X, Y) whose distribution is described by the next table:

X \ Y	2	4
1	a	0.1
2	0.1	0.3
3	a	$3a$

   (a) Find $a \in \mathbb{R}$ such that the above table is associated to a bidimensional random vector.

   (b) Write the marginal distributions.

   (c) Compute the probabilities $\mathbb{P}(X \geq 2,\, Y \geq 3)$, $\mathbb{P}(Y \leq 3 |\, X \geq 2)$ and $F_{(X,Y)}(2,\, 3)$ (where $F_{(X,Y)}$ is the cumulative distribution function associated to (X, Y)).

   (d) Find (the table of) the random variable $W = 2X - 0.5Y + 1$.

   (d) Check if the random variables X and Y are independente.

   Hint: the notation $\mathbb{P}(A|\, B)$ stands for conditional probability.