

LINEAR SYSTEMS

1 Solve the system

$$\begin{cases} 3x + 2y + 2z = 7 \\ x - y + 2z - t = 2 \\ 2x + y - z + t = 3 \\ x + y + z + t = 6 \end{cases}.$$

2 Solve the system

$$\begin{cases} 2x + y - z + 2t = -3 \\ 4x - 3y + 5z - 4t = 5 \\ x - 2y + 3z - 3t = 4 \end{cases}.$$

3 Solve the system

$$\begin{cases} x - 3y + z - t = 0 \\ 2x + y - z + 2t = 0 \end{cases}.$$

4 Solve the system

$$\begin{cases} x + 5y + 4z - 13t = 3 \\ 3x - y + 2z + 5t = 2 \\ 5x + y + 5z + t = 3 \end{cases}.$$

5 Solve the system

$$\begin{cases} x + y - 2z = -11 \\ 4x + 2y + z = 7 \\ 7x + 4y + z = 7 \end{cases}.$$

6 Discuss and solve the system

$$\begin{cases} ax + y + z = 0 \\ x + y - 2z = 0, \quad a \in \mathbb{R}. \\ x + y + az = 0 \end{cases}$$

7 Find $\lambda \in \mathbb{R}$ so that the system

$$\begin{cases} (1 + \lambda)x + y + z + t = 0 \\ x + (1 + \lambda)y + z + t = 0 \\ x + y + (1 - \lambda)z + t = 0 \\ x + y + z + (1 - \lambda)t = 0 \end{cases}$$

has nonzero solutions.

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8 Discuss and solve the system

$$\begin{cases} x + y - z + 2t = 0 \\ ax + y + z + t = 0 \\ x - y + 3z - 3t = 0 \\ 4x + 2y + at = 0. \end{cases}, \quad a \in \mathbb{R}.$$

9 Discuss and solve the system

$$\begin{cases} x + y + z + t = 10 \\ x + y - z - 2t = -7 \\ 5x + 5y - z + at = -4 \\ x + y + 3z + 4t = 28 \end{cases}, \quad a \in \mathbb{R}.$$

10 Prove that if $a, b, c \in \mathbb{Z}$, then the system

$$\begin{cases} ax + by + cz = \frac{1}{2}x \\ cx + ay + bz = \frac{1}{2}y \\ bx + cy + az = \frac{1}{2}z \end{cases}$$

has only the null solution.

11 Solve the matrix system

$$\begin{cases} -2X + Y = A \\ 3X - 2Y = B \end{cases}, \quad A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}.$$