Seminar 11, Analiză matematică, semestrul I, 2013–2014

1 Integrale curbilinii

1.1 Integrale curbilinii de speţa I

Exerciţiul 1.1 *Calculaţi următoarele integrale:*

1. $\int\limits_C xyds$, cu $(C): \begin{cases} x = t \\ y = t^2 \end{cases}, \ t \in [-1, 1]$.

2. $\int\limits_C z(x^2 + y^2)ds$, cu $(C): \begin{cases} x = t \cdot \cos t \\ y = t \cdot \sin t \\ z = t \end{cases}, \ t \in [-1, 1]$.

3. $\int\limits_C y \cdot e^{-x}ds$, cu $(C): \begin{cases} x = \ln(1 + t^2) \\ y = 2 \operatorname{arctg} t - t + 1 \end{cases}, \ t \in [0, 1]$.

4. $\int\limits_C yds$, cu $(C): \begin{cases} x = r \cdot (t - \sin t) \\ y = r \cdot (1 - \cos t) \end{cases}, \ t \in [0, 2\pi]$.

5. $\int\limits_C (x^2 + y^2) \cdot \ln zds$, cu $(C): \begin{cases} x = e^t \cdot \cos t \\ y = e^t \cdot \sin t \\ z = e^t \end{cases}, \ t \in [0, 1]$. .

6. $\int\limits_C (x + y + z)ds$, cu $(C): \begin{cases} x = a \cdot \cos t \\ y = a \cdot \sin t \\ z = bt \end{cases}, \ t \in \left[0, \frac{\pi}{2}\right]$.

7. $\int\limits_C ds$, cu $(C): \begin{cases} x = t \cdot \arcsin t + \sqrt{1 - t^2} \\ y = t - \sqrt{1 - t^2} \arcsin t \end{cases}, \ t \in [-1, 1]$.

8. $\int\limits_C ds$, cu $(C): \begin{cases} x = t \\ y = \sqrt{2} \cdot \ln(\cos t) \\ z = \operatorname{tg} t - t \end{cases}, \ t \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

9. $\int\limits_C xyds$, cu $(C): \begin{cases} x = a \cdot \cos t \\ y = b \cdot \sin t \end{cases}, \ t \in \left[0, \frac{\pi}{2}\right]$.

10. $\int\limits_C xy^2zds$, cu $(C): \begin{cases} x = t \\ y = \frac{1}{3}\sqrt{8t^3} \\ z = \frac{1}{2}t^2 \end{cases}, \ t \in [0, 1]$.

11. $\int\limits_C \sqrt{x^2 + y^2}ds$, cu $(C): \begin{cases} x = \cos t + t \cdot \sin t \\ y = \sin t - t \cdot \cos t \end{cases}, \ t \in [0, 2\pi]$.

12. $\int\limits_C \frac{ds}{x^2 + y^2 + z^2}$, cu $(C): \begin{cases} x = \cos t \\ y = \sin t \\ z = t \end{cases}, \ t \in [0, 2\pi]$.

13. $\int\limits_C \sqrt{x^2 + y^2}ds$, cu $(C): \begin{cases} x = 1 + \cos t \\ y = \sin t \end{cases}, \ t \in [0, 2\pi]$.

1.2 Integrale curbilinii de speţa a II-a

Exerciţiul 1.2 *Calculaţi următoarele integrale:*

1. $\int\limits_C xdx - ydy$, cu $(C) : \begin{cases} x = e^t \\ y = e^{-t} \end{cases}, \; t \in [0,1]$.

2. $\int\limits_C xydx - y^2dy$, cu $(C) : \begin{cases} x = t^2 \\ y = t^3 \end{cases}, \; t \in [0,1]$.

3. $\int\limits_C \sqrt{yz}dx + \sqrt{xz}dy + \sqrt{xy}dz$, cu $(C) : \begin{cases} x = t \\ y = t^2 \\ z = t^3 \end{cases}, \; t \in [0,1]$.

4. $\int\limits_C xdx - xydy + xyzdz$, cu $(C) : \begin{cases} x = e^t \\ y = e^{-t} \\ z = \sqrt{2} \cdot t \end{cases}, \; t \in [0,1]$.

5. $\int\limits_C (x-y)dx - (x+y)dy$, cu $(C) : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

6. $\int\limits_C z \cdot \sqrt{a^2 - x^2}dy + (x^2 + y^2)dz$, cu $(C) : \begin{cases} x = a \cdot \cos t \\ y = a \cdot \sin t \\ z = bt \end{cases}, \; t \in \left[0, \dfrac{\pi}{2}\right]$.

7. $\int\limits_{(0,0)}^{(2,1)} 2xydx + x^2dy$. 8. $\int\limits_{(0,2)}^{(2,0)} y^2e^x dx + 2ye^x dy$.

9. $\int\limits_{(1,1,1)}^{(a,b,c)} yzdx + xzdy + xydz$. 10. $\int\limits_{(1,0,-3)}^{(6,4,8)} xdx + ydy - zdz$.

11. $\int\limits_{\overset{\frown}{OA}} (x+y)dx + (x-y)dy$; $O(0,0),\ A(\pi,\pi)$ pe $\begin{cases} \text{segmentul } [OA] \\ y = x + \sin x \\ y = \frac{1}{\pi} \cdot x^2 \end{cases}, \; t \in \left[0, \dfrac{\pi}{2}\right]$.

12. $\int\limits_{(1,1,1)}^{(2,2,3)} \left(x^2 - yz + \dfrac{y}{x^2+y^2}\right) dx + \left(y^2 - zx - \dfrac{x}{x^2+y^2}\right) dy + \left(z^2 - xy\right) dz$.

13. $\int\limits_C ydx - xdy + \left(x^2 + y^2 + z^2\right) dz$, cu $(C) : \begin{cases} x = -t\cos t + \sin t \\ y = t \sin t + \cos t \\ z = t + 1 \end{cases}, \; t \in [0, \pi]$.

14. $\int\limits_{(-1,3,1)}^{(2,6,3)} \dfrac{y}{z}dx + \dfrac{x}{z}dy - \dfrac{xy}{z^2}dz, \quad z \neq 0$.

Exerciţiul 1.3 *Determinaţi aria cercului şi aria elipsei cu ajutorul integralei curbilinii, folosind formula:*

$$\mathcal{A} = \frac{1}{2} \oint\limits_C x \cdot dy - y \cdot dx.$$