SEMINAR 2, Analiză matematică, semestrul I, 2014–2015

Limite fundamentale

- $\lim\limits_{\substack{n\to\infty\\ x_n\to\infty}} \left(1+\dfrac{1}{x_n}\right)^{x_n} = e.$
- $\lim\limits_{\substack{n\to\infty\\ x_n\to 0}} (1+x_n)^{\frac{1}{x_n}} = e.$
- $\lim\limits_{\substack{n\to\infty\\ x_n\to 0}} \dfrac{\sin x_n}{x_n} = 1.$
- $\lim\limits_{\substack{n\to\infty\\ x_n\to 0}} \dfrac{\ln(1+x_n)}{x_n} = 1.$
- $\lim\limits_{\substack{n\to\infty\\ x_n\to 0}} \dfrac{a^{x_n}-1}{x_n} = \ln a.$

Exerciţiul 2.1. Să se calculeze următoarele limite de şiruri:

a) $\lim\limits_{n\to\infty} \left(\dfrac{n+2}{n+1}\right)^{3n+1}$

b) $\lim\limits_{n\to\infty} \left(\dfrac{n-1}{n}\right)^{2n+1}$

c) $\lim\limits_{n\to\infty} \left(\dfrac{n+1}{n}\right)^{n+1} \cdot \left(\dfrac{3n+1}{3n}\right)^{2n+1}$

d) $\lim\limits_{n\to\infty} \left(\sqrt{n^2+n}-n\right)^n$

e) $\lim\limits_{n\to\infty} \left(\sqrt{n^2+2n}-n\right)^n$

f) $\lim\limits_{n\to\infty} \left(\sqrt{n^2+3n}-n\right)^n$

g) $\lim\limits_{n\to\infty} \left(\dfrac{\sqrt{n^2+1}-n}{\sqrt{n^2+2}-n}\right)^n$

h) $\lim\limits_{n\to\infty} \left(\dfrac{\sqrt{n}+\sqrt{n+3}}{2\sqrt{n+1}}\right)^n$

i) $\lim\limits_{n\to\infty} \left(\dfrac{n^2+n+1}{n^2+n+2}\right)^{n^2+1}$

j) $\lim\limits_{n\to\infty} \left(\sqrt[3]{n^3+3n^2}-n\right)^{n^2}$

Exerciţiul 2.2. Să se calculeze următoarele limite de şiruri:

a) $\lim\limits_{n\to\infty} \dfrac{2n^2+3}{3n+1} \sin \dfrac{n}{n^2+1}$

b) $\lim\limits_{n\to\infty} (3n+1)^3 \sin \dfrac{1}{n} \operatorname{tg} \dfrac{1}{n^2}$

c) $\lim\limits_{n\to\infty} \dfrac{\sin \frac{\alpha}{n}}{\sin \frac{\beta}{n}}$

d) $\lim\limits_{n\to\infty} \dfrac{2n^2+1}{3n+1} \sin \dfrac{2n^2}{1^2+2^2+...+n^2}$

e) $\lim\limits_{n\to\infty} (5n^2+3) \sin \dfrac{1}{1+2+...+n}$

f) $\lim\limits_{n\to\infty} (-2n+1) \operatorname{tg} \dfrac{n}{n^2+1}$

g) $\lim\limits_{n\to\infty} \dfrac{\sin n}{n}$

h) $\lim\limits_{n\to\infty} \dfrac{\operatorname{arctg} n}{n}$

Exerciţiul 2.3. Să se calculeze următoarele limite de şiruri:

a) $\lim\limits_{n\to\infty} n\left(3^{\frac{1}{n}}-1\right)$

b) $\lim\limits_{n\to\infty} n^2\left(5^{\frac{1}{n}}-1\right)$

c) $\lim\limits_{n\to\infty} n^2\left(2^{\frac{1}{n(n+1)}}-1\right)$

d) $\lim\limits_{n\to\infty} n\left(2^{\frac{2n}{n^2+3n)}}-1\right)$

e) $\lim\limits_{n\to\infty} \dfrac{\sqrt[n]{8}-1}{\sqrt[n]{5}-1}$

f) $\lim\limits_{n\to\infty} \dfrac{\sqrt[n]{9}-\sqrt[n]{5}}{\sqrt[n]{4}-\sqrt[n]{3}}$

g) $\lim\limits_{n\to\infty} \dfrac{\sqrt[n]{9}-\sqrt[2n]{5}}{\sqrt[3n]{4}-\sqrt[4n]{3}}$

Exerciţiul 2.4. Să se calculeze următoarele limite de şiruri:

a) $\lim\limits_{n\to\infty} \dfrac{\ln\left(1+e^{3n}\right)}{\ln\left(1+e^{2n}\right)}$

b) $\lim\limits_{n\to\infty} \dfrac{\ln\left(n^2+2n+4\right)}{\ln\left(n^8-n+3\right)}$

c) $\lim\limits_{n\to\infty} \sqrt[3]{n}\mathrm{tg}\dfrac{1}{\sqrt{n}}$

d) $\lim\limits_{n\to\infty} n\sqrt{n}\left(e^{\frac{\sqrt{n}}{n(n+1)}-1}\right)$

e) $\lim\limits_{n\to\infty} ()^{\sqrt{n^2+1}-\sqrt{n}}$

f) $\lim\limits_{n\to\infty} \left(1+\sin\dfrac{1}{n}\right)^n$

Exerciţiul 2.5. Să se calculeze următoarele limite de şiruri:

a) $\lim\limits_{n\to\infty} \dfrac{1+\sqrt{2}+\sqrt{3}+...+\sqrt{n}}{n\sqrt{n}}$

b) $\lim\limits_{n\to\infty} \dfrac{1^p+2^p+...+n^p}{n^{p+1}}$