

## 1 Serii numerice

**Exercițiul 1.1** *Studiați convergența seriilor:*

I. Criteriul comparației cu limită:

$$\begin{aligned}
 & 1. \sum_{n=1}^{\infty} \frac{-2n+1}{n^2 \sqrt[3]{n}+7} \quad 2. \sum_{n=1}^{\infty} [\ln(3^n+7^n) - n \ln 7] \\
 & 3. \sum_{n=1}^{\infty} \frac{1}{n^2+2} \quad 4. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}} \quad 5. \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{\sqrt{n}} \\
 & 6. \sum_{n=1}^{\infty} \frac{1}{n} \ln \frac{n+1}{n} \quad 7. \sum_{n=1}^{\infty} \frac{n+3}{n^2+n+8} \quad 8. \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}+2n+1} \\
 & 9. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+7n^2+2}} \quad 10. \sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n^2+2}}
 \end{aligned}$$

II. Criteriul rădăcinii:

$$\begin{aligned}
 & 11. \sum_{n=1}^{\infty} \left( \frac{3n^3+4n+5}{3n^3+4n+6} \right)^{n^4} \quad 12. \sum_{n=1}^{\infty} \left( \sqrt{n^2+2n+5} - n \right)^{n^2} \\
 & 13. \sum_{n=1}^{\infty} \left( \frac{n+1}{3n+2} \right)^n \quad 14. \sum_{n=1}^{\infty} \left( \frac{4n+1}{9n+2} \right)^{\frac{n}{2}} \quad 15. \sum_{n=1}^{\infty} \left( \frac{n}{2n+1} \right)^{n^2} \\
 & 16. \sum_{n=1}^{\infty} \left( \frac{2n+5}{2n+6} \right)^{n^2} \quad 17. \sum_{n=1}^{\infty} \left( \frac{n^2+2n+5}{n^2+2n+7} \right)^{n^3} \quad 18. \sum_{n=1}^{\infty} \left( \sqrt[3]{n^3+4n^2+5} - n \right)^n
 \end{aligned}$$

III. Criteriul raportului:

$$\begin{aligned}
 & 17. \sum_{n=1}^{\infty} \left[ \frac{n! \cdot 2^n}{1 \cdot 4 \cdot 7 \cdot \dots \cdot (3n-2)} \right]^2 \\
 & 18. \sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot \dots \cdot (3n-1)}{1 \cdot 5 \cdot \dots \cdot (4n-3)} \quad 19. \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n-1)} \\
 & 20. \sum_{n=1}^{\infty} \frac{3 \cdot 7 \cdot 11 \cdot \dots \cdot (4n-1)}{2^n \cdot (n+1)!} \quad 21. \sum_{n=1}^{\infty} \frac{3^{n+1} \cdot n!}{4^n \cdot 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n+1)} \\
 & 22. \sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot \dots \cdot (3n-1)}{1 \cdot 5 \cdot \dots \cdot (4n-3)} \cdot \frac{1}{2n+1} \quad 23. \sum_{n=1}^{\infty} 3^n \sin \frac{1}{n} \quad 24. \sum_{n=1}^{\infty} \operatorname{tg} \frac{\pi}{4^n}
 \end{aligned}$$

IV. Criteriul Raabe-Duhamel:

$$\begin{aligned}
 & 24. \sum_{n=1}^{\infty} \frac{2^n \cdot n!}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} \cdot \frac{1}{\sqrt{2n+1}} \\
 & 25. \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^n \cdot n!} \quad 26. \sum_{n=1}^{\infty} \left[ \frac{n! \cdot 2^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} \right]^2
 \end{aligned}$$

**Exercițiul 1.2** *Studiați absoluta convergență și semiconvergența următoarelor serii:*

$$1. \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2+n+1}. \quad 2. \sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^4+n^2+1}.$$