Seminar 4, Analiză matematică, semestrul I, 2014–2015

1 Limite şi continuitate

Exerciţiul 1.1 *Să se calculeze limitele:*

1. $\lim\limits_{x\to -2} \dfrac{x^3+5x^2+4x-4}{x^2+4x+4}$. 2. $\lim\limits_{x\to 1} \dfrac{\sqrt{5x-1}-\sqrt{3x+1}}{x^2-3x+2}$.
3. $\lim\limits_{x\to \infty} \dfrac{-5x^4+3x^2-2}{4x^4+x+1}$. 4. $\lim\limits_{x\to -\infty} \dfrac{x^3+1}{-6x^3-3x^2-x+2}$.
5. $\lim\limits_{x\to \infty} \dfrac{x-1}{x^2+5x-3}$. 6. $\lim\limits_{x\to -\infty} \dfrac{4x^3+3}{2x^5+4}$.
7. $\lim\limits_{x\to \infty} \dfrac{x^3+3x^2+3x-1}{7x+1}$. 8. $\lim\limits_{x\to -\infty} \dfrac{6x^6+4x^4+2x-1}{x^2+2}$.
9. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}+3\sqrt{x}-4}{x-1}$. 10. $\lim\limits_{x\to -\infty} (\sqrt{x^2-3x+5}+x)$.
11. $\lim\limits_{x\to \infty} \left(\dfrac{x^2+2x+3}{x^2-x+1}\right)^x$. 12. $\lim\limits_{x\to -\infty} \left(\dfrac{3x^3+x+3}{3x^3+x^2+1}\right)^{\frac{5x^2-1}{-x+3}}$.
13. $\lim\limits_{x\to -\infty} x\ln(\sqrt{x^2+2x+7}+x)$. 14. $\lim\limits_{x\to 0} \dfrac{\sin 5x-\sin 3x}{x}$.
15. $\lim\limits_{x\to 0} \dfrac{\cos 3x-\cos 9x}{x\sin 5x}$. 16. $\lim\limits_{x\to 0} \dfrac{\operatorname{tg}(\operatorname{tg} x)-\sin(\sin x)}{\operatorname{tg} x-\sin x}$.
17. $\lim\limits_{x\to 0} \dfrac{\cos(\operatorname{tg} x)-\cos(\sin x)}{x(\operatorname{tg} x-\sin x)}$. 18. $\lim\limits_{x\to 0} \dfrac{\ln(1+\sin 2x)}{\operatorname{tg} 3x}$.
19. $\lim\limits_{x\to \infty} \left(\dfrac{\ln(1+x)}{\ln x}\right)^{x\ln x}$. 20. $\lim\limits_{x\to \frac{\pi}{4}} (\operatorname{tg} x)^{\frac{1}{\sqrt{1-\sin 2x}}}$.
21. $\lim\limits_{x\to \frac{\pi}{6}} \left(\sqrt{3}\operatorname{tg} x\right)^{\operatorname{tg} 3x}$.

Exerciţiul 1.2 *Să se determine numerele reale a, b, c astfel încât să fie verificate relaţiile:*

1. $\lim\limits_{x\to -\infty} x\ln(\sqrt{x^2+bx+c}+ax) = 5$.
2. $\lim\limits_{x\to -\infty} \left(\dfrac{ax+18}{bx+3}\right)^{\dfrac{x+7}{cx+5}} = e^3$.

Exerciţiul 1.3 *Să se arate că funcţiile următoare nu au limită în punctele specificate:*

1. $f(x)=\sin\dfrac{1}{x}$, în $x_0=0$.
2. $f(x)=\cos x$, la $+\infty$.
3. $f(x)=e^x(1+3\cos x)$, la $+\infty$.
4. $f(x)=\dfrac{x\sin(\frac{1}{x})}{\ln(1+x)}$, în $x_0=0$.

Exerciţiul 1.4 *Să se studieze continuitatea funcţiilor. În caz de discontinuitate, precizaţi tipul acesteia:*

1. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} x\cos\frac{1}{x},\ x\neq 0\\ 0,\ x=0\end{cases}.$
2. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} \dfrac{\ln(1+\operatorname{arctg}3x)}{\sin 2x},\ x<0\\ \frac{3}{2},\ x=0\\ \frac{1}{\ln 5}\cdot\dfrac{5^{\sin(3x^2)}-1}{1-\cos 4x},\ x>0\end{cases}.$
3. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} \dfrac{1}{2+3^{\frac{1}{x-1}}},\ x\neq 1\\ 5,\ x=1\end{cases}.$
4. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} \dfrac{e^x\cos x-1}{\sin x},\ \text{dacă } x<0\\ a,\ \text{dacă } x=0\\ \dfrac{\ln(1+x)}{x},\ \text{dacă } x>0\end{cases}$, unde $a\in\mathbb{R}$.
5. $f(x)=\begin{cases} \operatorname{arctg}\frac{1}{x-1},\ x\neq 1\\ 4,\ x=1\end{cases}.$
6. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} \dfrac{\cos 3x-\cos 7x}{x^2},\ x<0\\ 3,\ x=0\\ \dfrac{\ln(1+3^x)}{x},\ x>0\end{cases}.$
7. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} 3-2x-x^2,\ x\in[0,1)\\ 1,\ x=1\\ x^{\frac{2}{x^2-1}},\ x>1\end{cases}.$
8. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} \left(\dfrac{x-4}{x-1}\right)^{1-x},\ x\in[0,1)\\ 1,\ x=1\\ \dfrac{\cos\frac{\pi x}{2}}{x-1},\ x>1\end{cases}.$
9. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} 1,\ x\in\mathbb{Q}\\ 2,\ x\in\mathbb{R}\setminus\mathbb{Q}\end{cases}.$
10. $f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases} x^2,\ x\in\mathbb{Q}\\ x+6,\ x\in\mathbb{R}\setminus\mathbb{Q}\end{cases}.$