Sample Subjects for the Semestrial Test - 12/8/14 (A)

TS − 1 Invert the matrix

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

and (then) solve the matrix equation

$$X \begin{bmatrix} -1 & 0 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & 4 \\ 1 & 3 & -1 \end{bmatrix}.$$

Hint. The equation is of the form $XA = B$ with the solution $X = BA^{-1}$. But this solution can also be found (under its transpose form) by applying the Gaussian elimination to the block matrix $[A^{\mathrm{T}} | B^{\mathrm{T}}] \rightarrow \ldots \rightarrow [I_3 | A^{-T}B^{\mathrm{T}} = X^{\mathrm{T}}]$.

TS − 2 Solve the homogeneous system

$$\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{1} & -1 \\ 0 & 3 & -1 & 1 \\ 2 & 7 & 1 & -1 \end{bmatrix} X = \mathbf{0}.$$

TS − 3 Determine the real parameter α so that the two vectors

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ \alpha \\ \alpha \end{bmatrix}, \ \mathbf{u}_2 = \begin{bmatrix} \alpha \\ 1 \\ 2\alpha - 1 \end{bmatrix}$$

be linearly dependent and find a linear dependence relation between them.

TS − 4 In a space $V = \mathcal{L}(A)$, $A = [a_1 \ a_2 \ a_3]$ is considered the vector $x = 2a_1 - a_2 + 5a_3$. The basis is changed for $B = [b_1 \ b_2 \ b_3]$ by the transformation matrix

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}.$$

It is required to find the coordinates X_B in the "new" basis and to check them.

TS − 5 Determine the dimensions of subspaces $U, W \subseteq \mathbb{R}^3$ respectively spanned by

$A : [2 \ 3 \ -1]^{\mathrm{T}}, [1 \ 2 \ 2]^{\mathrm{T}}, [1 \ 1 \ -3]^{\mathrm{T}}$ and

$B : [1 \ 2 \ 1]^{\mathrm{T}}, [1 \ 1 \ -1]^{\mathrm{T}}, [1 \ 3 \ 3]^{\mathrm{T}}$;

Then find the dimensions and a basis for each of $U + W$ & $U \cap W$.

Sample Subjects for the Semestrial Test - 12/8/14 (B)

TS − 1

Determine the rank of the matrix

$$A = \begin{bmatrix} -2 & 7 & 2 & -3 & 5 \\ 2 & 1 & -1 & 2 & 1 \\ 0 & 8 & 1 & -1 & 6 \end{bmatrix},$$

find a basis spanning COLSP_A and express the other columns in this basis.

TS − 2

Find the (values of) the real parameter m so that the linear system $A(m)X = b(m)$ be consistent and find its solution(s) in such cases :

$$A(m) = \begin{bmatrix} 1 & -m \\ 2 & 1 \\ 3 & m-1 \end{bmatrix}, \quad b(m) = \begin{bmatrix} -1 \\ m \\ -m+1 \end{bmatrix}.$$

TS − 3

A bilinear form $f : V \times V \to \mathbb{R}$ is defined by its matrix in a basis A, namely

$$f(A^{\mathrm{T}}, A) = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 3 & -5 & 4 \end{bmatrix}.$$

It is required to determine $\operatorname{rank} f$, its value $f(x, y)$ for $x = a_1 - 2a_2 + 3a_3$ and $y = 4a_1 + a_3$, and also the matrix of f in the basis B, where

$$B^{\mathrm{T}} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} A^{\mathrm{T}}.$$

TS − 4

A linear form $f : V \to \mathbb{R}$ is defined by its coefficients in a basis A, by $f(A) = [\alpha] = [8 \;\; -2 \;\; 1]$. It is required the value $f(3a_1 - 2a_2 + a_3)$

The basis is changed for $B = [b_1 \;\; b_2 \;\; b_3]$ by the transformation matrix

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 3 \\ -1 & 2 & 1 \end{bmatrix}.$$

Determine the coordinates X_B in basis B, $[\beta] = f(B)$ and $f(x)$ with basis B.

TS − 5

Determine the a basis for the subspace $U = S =$ the solution subspace to the H-system

$$\begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & 1 \\ 2 & -2 & 0 \end{bmatrix} X = \mathbf{0}. \quad W = \mathcal{L}([b_1 \;\; b_2]), \; b_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \;\&\; b_2 = \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix}.$$

Check *Grassmann's Theorem* for $U, W \subseteq_{\text{subsp}} \mathbb{R}^3$.

Sample Subjects for the Semestrial Test - 12/8/14 (C)

TS – 1 Turn the matrix, given below, into a quasi-triangular and quasi-diagonal form.

$$A = \begin{bmatrix} 2 & 1 & 3 & -1 \\ 3 & -1 & 2 & 0 \\ 1 & 3 & 4 & -2 \\ 4 & -3 & 1 & 1 \end{bmatrix}.$$

Identify a subset of independent columns and express the other columns in terms of the independent ones. Alternatively, solve the H-system $AX = \mathbf{0} \in \mathbb{R}^4$.

TS – 2 There are considered the three vectors in $\mathbb{R}^4$,

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -1 \\ 1 \\ -3 \\ 0 \end{bmatrix}.$$

It is required to find a basis and the dimension for $\mathcal{L}(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\})$.

TS – 3 A symmetric bilinear form $f : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ is defined by its matrix in the standard basis E, namely

$$f(E^{\mathrm{T}}, E) = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & -1 \\ 1 & -1 & 4 \end{bmatrix} \underset{\text{not}}{=} [\,\varepsilon\,].$$

It is required to determine the value $f(a_1, a_2)$ for $a_1 = [1 \ \ 2 \ -2]^{\mathrm{T}}$, $a_2 = [4 \ \ 0 \ \ 3]^{\mathrm{T}}$ and also a basis B for $U^{\perp f}$ where $U = \mathcal{L}(\{a_1, a_2\})$.

TS – 4 The linear form $f : \mathbb{R}^3 \to \mathbb{R}$ is defined by its analytic expression

$$f(X) = x_1 - 2x_2 + 4x_3.$$

It is required $f(X)$ for $X = [-4 \ \ 7 \ \ 2]^{\mathrm{T}}$, the coordinates X_A in the basis

$$A = [a_1 \ \ a_2] \ \ a_3]), \ a_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, a_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \ a_3 = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix};$$ check $f(X)$ with basis A.

TS – 5 Study the consistency of the three linear systems, $AX = [\,b^{(1)} \ \ b^{(2)} \ \ b^{(3)}\,]$ given by their augmented matrix

$$\tilde{A} = [\,A \mid B\,] = \left[\begin{array}{rrr|rrr} 2 & -1 & 0 & 3 & 4 & 0 \\ -1 & 1 & -1 & 2 & 1 & 0 \\ 3 & -1 & -1 & 8 & 3 & 0 \end{array}\right] X = \mathbf{0}.$$

Find the general solutions for the consistent systems.