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Constant Mean Curvature and Biharmonic Submanifolds
Principal Investigator: Assoc. Prof. Dorel Fetcu

Scientific Report
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The team members began the research activities related to our project in October 2015, aiming at attaining the proposed objectives.

Thus, we obtained our first results concerning one of the topics of the project's objective *Study of biharmonic and biconservative submanifolds in certain 3-dimensional spaces* in our paper

- D. Fetcu, S. Nistor, and C. Oniciuc, ***On biconservative surfaces in 3-dimensional space forms,***

accepted for publication by *Communications in Analysis and Geometry.*

Biconservative surfaces, i.e., those surfaces for which the tangent part of the bitension field vanishes, that do not have constant mean curvature, in space forms were studied by R. Caddeo, S. Montaldo, C. Oniciuc, and P. Piu, and, in a paper from 2014, they found the explicit equations of such surfaces in R^3, S^3, and H^3. In the same paper, they also proved that the Gaussian curvature of a biconservative surface satisfies an equation that looks very much like the equation used by G. Ricci-Curbastro, in 1895, to characterize minimal surfaces with negative Gaussian curvature in R^3 and then by H. B. Lawson, in 1970, to extend this result to constant mean surfaces in space forms. Surfaces satisfying this so called Ricci condition are also called Ricci surfaces and the metric on such a surface is a Ricci metric.

Encouraged by this similitude, we proved that there is a simple way to transform biconservative surfaces in R^3, S^3, and H^3, in Ricci surfaces. Moreover, in the case of biconservative surfaces in R^3 we explicitly found the Ricci metric on such a surface.

Whilst the Ricci condition characterizes intrinsically minimal surfaces in 3-dimensional space forms N^3(c), the similar condition found by Caddeo, Montaldo, Oniciuc, and Piu, for biconservative surfaces fail to do the same in their case. In our paper, we also found the intrinsic necessary and sufficient conditions for a Riemannian surface to be locally embedded in N3(c) as a biconservative surface without constant mean curvature.

These results were presented by C. Oniciuc at the scientific symposium “Al. I. Cuza University's Days” (October 23, 2015) and by S. Nistor at the international conference “Applied and Pure Mathematics (ICAPM 2015)” (November 6-8, 2015), at the Gheorghe Asachi Technical University of Iasi.

Another article, which is still a work in progress and will be authored by S. Nistor, devoted to the study of biconservative surfaces is

- ***Some remarks on biconservative surfaces in 3-dimensional space forms.***

In this paper, the author studies some of the global properties of Riemannian surfaces which admit biconservative immersions in three-dimensional space forms, i.e., in R^3, S^3, and H^3. More exactly, she investigates the completeness and simply connectedness of such surfaces.

In this phase of the project we did not make any mobility and logistic expenses. The initial budget breakdown for the year 2015 was observed.

Accepted papers:

1. D. Fetcu, S. Nistor, and C. Oniciuc, ***On biconservative surfaces in 3-dimensional space forms,*** Comm. Anal. Geom., to appear.

Work in progress:

1. S. Nistor, ***Some remarks on biconservative surfaces in 3-dimensional space forms.***

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