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Constant Mean Curvature and Biharmonic Submanifolds

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Scientific Report 2016

During the second year of our project the team members continued their research activities within the framework designed by our objectives.

Articles

A paper studying topics of the project's objectives **O1**. *Study of submanifolds with parallel mean curvature vector field in Riemannian manifolds* and **O2**. *Study of biharmonic and biconservative submanifolds in certain 3-dimensional spaces* is

- E. Loubeau and C. Oniciuc, ***Constant mean curvature proper-biharmonic surfaces of constant Gaussian curvature in spheres***, J. Math. Soc. Japan 68 (2016), 997–1024.

In this article constant mean curvature (CMC) surfaces (a more general case than that of parallel mean curvature surfaces) in spheres are investigated under the extra condition of biharmonicity. Using a classification of finite type surfaces in spheres given by Myata, the authors obtain a complete description of such immersions and show that for any given number h between 0 and 1 there exist CMC proper-biharmonic planes and cylinders in the fifth dimensional Euclidean sphere with mean curvature h , and also find a necessary and sufficient condition on h for the existence of CMC proper-biharmonic tori in this sphere. The paper ends with results on flat CMC surfaces in general n -dimensional Euclidean spheres.

We continued, from a new point of view, this study in our work

- D. Fetcu, E. Loubeau, and C. Oniciuc, ***Biharmonic tori in spheres***, preprint 2016,

where it is considered the case of proper-biharmonic tori with constant mean curvature in spheres. We propose a different approach from that in Loubeau and Oniciuc's paper and ask whether a given torus admits a full CMC proper-biharmonic immersion in a Euclidean sphere. We develop a study for two

families of tori: rectangular tori with a side of length equal to one and square tori. In the first case we determine the expressions of all such rectangular tori that admit full CMC proper-biharmonic immersions in an n -dimensional unit Euclidean sphere, as well as these immersions. It turns out that the only admissible dimensions for the ambient space are 5 and 7. The case of square tori is more flexible and we prove that there are examples of full CMC proper-biharmonic immersions of such tori in spheres with any odd dimension greater or equal to 5.

Another research direction approached by one of our team members is related to the objective **O6. Study of magnetic curves in product spaces**. The result of this study is the article

- S. L. Druță-Romaniuc, J. I. Inoguchi, M. Munteanu, and A. I. Nistor, *Magnetic curves in cosymplectic manifolds*, Rep. Math. Phys. 78 (2016), 33 – 48.

In this paper are classified magnetic trajectories with respect to contact magnetic fields in cosymplectic manifolds of arbitrary dimension. It is also obtained a classification of Killing magnetic curves in product spaces $M \times \mathbb{R}$, where M is an arbitrary 2-dimensional Riemannian manifold, and a reduction theorem for magnetic curves in cosymplectic space forms of type $M(k) \times \mathbb{R}$, where $M(k)$ is a complex space form with constant holomorphic sectional curvature k . More precisely, it is shown that such a curve lies in the 3-dimensional space $M(k) \times \mathbb{R}$, this time $M(k)$ being a 2-dimensional complex space form.

Results related to the objective **O5. Study of equivariant biharmonic maps** were obtained in the article

- S. Montaldo, C. Oniciuc, and A. Ratto, *Reduction methods for the bienergy*, Rev. Roumaine Math. Pures Appl., to appear.

Ideas introduced in a paper from 2013 authored by the first and third author are developed here, in an article that focuses on reduction methods (group actions or, more generally, symmetries) for the bienergy. These techniques prove to be very useful in order to obtain examples of critical points of the bienergy by reducing the study of the relevant fourth order PDE's system to ODE's. In particular, there are studied rotationally symmetric biharmonic conformal diffeomorphisms between models. The reduction method is then applied to study an ample class of G -invariant immersions into the Euclidean space. Unlike in the case of harmonic maps and, in particular, in that of minimal immersions, that inspired this kind of studies, the use of reduction techniques in the study of biharmonicity is not yet very developed, but the examples provided by this paper suggest that they can be of further use in this case too.

One of the PhD Students working in our team, Simona Nistor, brought an important contribution to the fulfillment of the tasks raised by objective **O2. Study of biharmonic and biconservative submanifolds in certain 3-dimensional spaces** through the paper

- S. Nistor, *Complete biconservative surfaces in R^3 and S^3* , J. Geom. Phys. 110 (2016), 130 – 153.

The paper is concerned with complete biconservative surfaces in 3-dimensional Euclidean space and 3-dimensional unit Euclidean sphere. Biconservative surfaces in 3-dimensional space forms are characterized by the fact that the gradient of their mean curvature function is an eigenvector of the shape operator, and the author studies local and global properties of such surfaces with non-constant mean curvature function. Simply connected complete Riemannian surfaces that admit biconservative immersions in R^3 and S^3 are determined and, moreover, explicitly described.

Mobilities

Between 2 and 4 June 2016, Cezar Oniciuc attended the conference "International Workshop on Theory of Submanifolds" organized in Istanbul, Turkey, by the Istanbul Technical University. As an invited speaker he presented the talk entitled "*Biconservative Surfaces*". He also acted as a chairperson for one of the conference's sessions.

Between 3 and 8 June 2016, Dorel Fetcu attended "The XVIII-th International Conference Geometry, Integrability and Quantization" in Varna, Bulgaria. He presented the plenary talk "*Biharmonic Surfaces with Parallel Mean Curvature in Complex Space Forms*" and also acted as a chairperson for one of the sessions.

Dorel Fetcu, Simona Nistor, and Cezar Oniciuc attended the conference "Differential Geometry and Its Applications" organized in Brno, Czechia, by the Masaryk University between 11 and 15 July 2016. Dorel Fetcu presented the talk "*On Biconservative Surfaces*" and Simona Nistor the poster "*Global Properties of Biconservative Surfaces*".

Invited by Eric Loubeau, Simona Nistor and Cezar Oniciuc spent one week, between 29 October and 5 November 2016, at the Department of Mathematics of UBO (University of Western Brittany) in Brest, France. There, together with their host, they worked on older common projects (like the Fetcu, Loubeau, Oniciuc paper mentioned above) and have also begun a couple of new ones, related to our project.

ISI published papers

1. E. Loubeau and C. Oniciuc, *Constant mean curvature proper-biharmonic surfaces of constant Gaussian curvature in spheres*, J. Math. Soc. Japan 68 (2016), 997–1024.
2. S. L. Druță-Romaniuc, J. I. Inoguchi, M. Munteanu, and A. I. Nistor, *Magnetic curves in cosymplectic manifolds*, Rep. Math. Phys. 78 (2016), 33 – 48.
3. S. Nistor, *Complete biconservative surfaces in R^3 and S^3* , J. Geom. Phys. 110 (2016), 130 – 153.

Accepted papers

1. S. Montaldo, C. Oniciuc, and A. Ratto, *Reduction methods for the bienergy*, Rev. Roumaine Math. Pures Appl., to appear.

Preprints

1. D. Fetcu, E. Loubeau, and C. Oniciuc, *Biharmonic tori in spheres*, preprint 2016.

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