

## Exercises

1. (a) Jim takes an oral exam in *Probability* by answering to the questions written on an examination card. There are 20 such examination cards and Jim will receive one of them drawn at random. Of the 20 there are 8 favorable cards. Jim will get an A if he answers to the questions on the card correctly. What is the probability that Jim gets an A if he draws first a card ? But if is the second who draw a card ? But if he is the third ?

(b) Starting at a fixed time, we observe the gender of each newborn child at a certain hospital until a boy is born. We suppose that the probability to born a boy is  $p$ . Assume that the successive births are independent. Find the distribution of the random variable  $X$  whose values are the total number of the births observed.

Also find the cumulative distribution function associated to  $X$  and compute the probability of having to examine at most five births to see the first boy.

Compute the mean  $\mathbb{E}\left(\frac{1}{1+X}\right)$ .

2. Let  $p \in (0, 1)$ . Let the independent random variables  $X_i$  given by  $\mathbb{P}(X_i = 0) = q$  and  $\mathbb{P}(X_i = 2) = p$ , with  $i \in \mathbb{N}^*$ .

(a) Find the mean and the variance of  $X_i$ .

(b) Find the type of the distribution of the r.v.  $X_i/2$  and compute the variance of this r.v..

(c) Find the type of the distribution of the r.v.  $\bar{X}_k := \frac{\sum_{i=1}^k X_i}{2}$ .

3. Let  $X$  and  $Y$  be two random variables with the distributions

$$\mathbb{P}(X = -3) = a, \quad \mathbb{P}(X = -2) = 5/32, \quad \mathbb{P}(X = -1) = b, \quad \mathbb{P}(X = 0) = 5/16$$

and

$$\mathbb{P}(X = 1) = c, \quad \mathbb{P}(X = 2) = 1/32.$$

We know that

$$\mathbb{E}(X) = -1/2, \quad \text{Var}(X) = 5/4.$$

(a) Find  $a, b, c$ .

(b) Compute  $\mathbb{E}(3 + 2X)$ ,  $\mathbb{E}\left[(3 + 2X)^2\right]$ ,  $\text{Var}(3 + 2X)$ .

(c) Find the law (the table) of the r.v.  $Y := X^2$  and  $Z := |X|$ .